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QUANTUM ORIGIN OF THE UNIVERSE

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1. INTRODUCTION

The idea that the universe was created from nothing is at least as old as the Old Testament. The first serious scientific discussion of this possibility came several thousand years later. In 1973 Tryon[1] pointed out that all strictly conserved quantum numbers of a closed universe can be equal to zero, and so the whole universe can be a vacuum fluctuation. More recently, Brout et al.[2], Atkatz and Pagels[3], and Gott[4], have discussed cosmological models in which the role of the big bang is played by a quantum event due to some sort of a quantum instability of a simple initial state (flat space[2], closed static universe[3], or de Sitter space[4]). A weak point of this picture is that the universe could not stay in its initial state indefinitely long if that state was metastable. [The same problem arises in the biblical version of the creation: "What was God doing before He made heaven and earth? If He was at rest and doing nothing, why did He not continue to do nothing for ever after, as for ever before?"[5]]*) In this paper, I shall discuss a model[6] in which the universe is created by quantum tunneling from "nothing", where by "nothing" I mean a state with no classical space time. A short version of this paper has been published in Ref. 7.

2. BUBBLE NUCLEATION

Let us first consider a more familiar example of quantum tunneling: bubble nucleation in a metastable vacuum[8,9]. Suppose the surface rest energy of the bubble is σ and that the energy density of the true vacuum inside the bubble is smaller than that of the false vacuum outside by amount ε. Bubble formation does not change the energy of the system, thus

$$\Delta E = 4\pi\sigma R^2(1-\dot{R}^2)^{-1/2} - \frac{4\pi}{3}\varepsilon R^3 = 0 \tag{1}$$

Here, $R(t)$ is the radius of the bubble. The solution of Eq. (1) is

*)St. Augustine, who spent many years thinking about this problem, came up with a resolution which is similar to that suggested in the present paper. He concluded that before the universe was created, there was no time, and thus the question of what God was doing before is meaningless.[5]

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$$R = (R_o^2 + t^2)^{1/2}, \tag{2}$$

where $R_o = 3\sigma/\varepsilon$. It describes a bubble which contracts at $t < 0$, then bounces at a minimum size R_o, and expands at $t > 0$. In the actual history of the bubble the $t < 0$ part is absent: the bubble tunnels quantum-mechanically from $R = 0$ to $R = R_o$, and then evolves according to Eq. (2) with $t > 0$.

The tunneling probability can be written as

$$P = Ae^{-B} \tag{3}$$

In the semiclassical approximation, the exponent B can be found by integrating the absolute value of the canonical momentum, p, over the classically forbidden region $0 < R < R_o$:

$$B = 2 \int_o^{R_o} |p| dR \tag{4}$$

The Lagrangian corresponding to the energy (1) is

$$L = (4\pi/3)\varepsilon R^3 - 4\pi\sigma R^2(1 - \dot{R}^2)^{1/2}, \tag{5}$$

and the momentum is

$$p = (\partial L/\partial \dot{R}) = 4\pi\sigma R^2\dot{R}(1-\dot{R}^2)^{-1/2} = 4\ \pi\sigma R^2(1-R^2/R_o^2)^{1/2}, \tag{6}$$

where in the last equality I have used Eq. (1). From (4) and (6), we find

$$B = 8\pi\sigma \int_o^{R_o} (1-R^2/R_o^2)^{1/2}\ R^2 dR = \frac{27\pi^2}{2}\ \frac{\sigma^4}{\varepsilon^3}\ . \tag{7}$$

This simple-minded analysis of bubble nucleation is due to Voloshin et al.[8]. It was later extended by Coleman and others[9]. In particular, Coleman has shown that Eq. (7) is valid only if the thickness of the bubble wall is much smaller than R_o and that, in the general case, B can be obtained by finding a solution of the Euclidean field equations which approaches the false vacuum at space-time infinity. Such solutions are called bounce solutions, or instantons. The quantity B is given by

$$B = S_E, \tag{8}$$

where S_E is the Euclidean action of the instanton. The pre-exponential factor A can be found by considering small fluctuations around the instanton solution.

If we imagine that the surface of the bubble is inhabited by 2-dimensional creatures, then they will find themselves living in a (2+1)-dimensional de Sitter space,

$$ds^2 = d\tau^2 - R_o^2\ \cosh^2(\tau/R_o)(d\theta^2+\sin^2\theta d\phi^2) \tag{9}$$

where τ is the proper time on the bubble wall. If they are smart enough, they may also conclude that their universe was spontaneously created at $\tau = 0$, and so Eq. (9) applies only for $\tau > 0$. Now I am going to argue that our own universe might have arisen in a very similar way. The main difference is that I will not have to postulate that our universe is a boundary between hypothetical multi-dimensional vacua. The whole discussion will be based on the laws of physics in (3+1)-dimensional space.

3. CREATION OF UNIVERSES FROM NOTHING

Let us consider a model of interacting gravitational and matter fields, where for simplicity the matter fields are represented by a single Higgs field ϕ with an effective potential $V(\phi)$. If $\phi = \eta$ is the true minimum of the effective potential, then we require that $V(\eta) \sim 0$, so that the cosmological constant is small today. Besides $\phi = \eta$, $V(\phi)$ can have other extrema. If $\phi = \phi_0$ is such an extremum, $V'(\phi_0) = 0$, then $\phi = \phi_0 = \text{const}$ is a solution of the classical equation of motion for ϕ:

$$\Box\phi + V'(\phi) = 0. \tag{10}$$

The vacuum energy density at $\phi = \phi_0$ will, in general, be nonzero (and positive): $\rho_v = V(\phi_0) > 0$.

Our model will be based on a solution of the combined Einstein and scalar field equations in which $\phi = \phi_0$ and the gravitational field is described by a closed Robertson-Walker metric:

$$ds^2 = dt^2 - a^2(t)[d\chi^2 + \sin^2\chi(d\theta^2+\sin^2\theta d\phi^2)]. \tag{11}$$

The scale factor $a(t)$ satisfies the equation

$$\dot{a}^2 + 1 = H^2a^2, \tag{12}$$

where

$$H = (8\pi G\rho_v/3)^{1/2}. \tag{13}$$

The solution of Eq. (12) is the de Sitter space,

$$a(t) = H^{-1}\cosh(Ht). \tag{14}$$

It describes a closed universe which contracts at $t < 0$, then "bounces" at a minimum size $a_{min} = H^{-1}$, and expands at $t > 0$.

This behavior is very similar to that of the bubble (see Eqs. (2), (9)), and we are led to assume that the origin of the universe might also be similar. Then the universe has emerged at the bounce point having a finite size ($a=H^{-1}$) and zero "velocity" ($\dot{a} = 0$); its following evolution is described by Eq. (5) with $t > 0$.

The semiclassical tunneling probability can be found from Eqs. (3), (4) with R replaced by a and R_0 replaced by H^{-1}. The momentum p is given by $p = \partial L/\partial\dot{a}$, where L is obtained from the action

$$S = \int L\, dt = \frac{3\pi}{4G} \int dt\, (-\dot{a}^2 a + a - H^2 a^3) \qquad (15)$$

This gives

$$B = 2 \int_0^{H^{-1}} |p|\, da = \frac{3\pi}{G} \int_0^{H^{-1}} (1-H^2a^2)^{1/2}\, a da = \frac{3}{8G^2\rho_v}. \qquad (16)$$

An alternative description of the tunneling process is given by the bounce solution of the Euclidean field equations. In our case, the bounce solution can be obtained by changing $t \to i\tau$ in Eq. (14) (τ is the Euclidean time):

$$a(\tau) = H^{-1} \cos(H\tau). \qquad (17)$$

It describes a four-sphere, S^4, of radius H^{-1}. This is the well-known de Sitter instanton[10]. S^4 is a compact space (the bounce solution (17) is defined only for $|\tau| < \pi/2H$) and does not have an asymptotic region, and thus the instanton can be interpreted as describing a tunneling to the de Sitter space (14) from nothing, where by "nothing", I mean a state with no classical spacetime[6].

The Euclidean action for the de Sitter instanton is[10]

$$S_E = -3/8G^2\rho_v \qquad (18)$$

and Eq. (8) would give B equal but opposite in sign to that of Eq. (16). As will be clear from the discussion in the next section, the correct answer is given by Eq. (16); that is, one has to use[7] $B = |S_E|$. Note that Eq. (8) is valid in the usual case of bubble nucleation, since in that case the Euclidean action is positive-definite. In Ref. 6 I was misled by Eq. (8) into the conclusion that the tunneling probability is $P \propto \exp(3/8G^2\rho_v)$. After this work was completed, I learned that Linde[11] has independently derived the expression for P with the right sign of B.

4. MINISUPERSPACE APPROACH

The problem of determining the tunneling amplitude can also be approached by solving the "Schroedinger equation" for the wave function of the universe. In the general case, the wave function $\Psi(g_{ij},\phi)$ is defined on a space of all possible 3-geometries and scalar field configurations (superspace). The role of the Schroedinger equation for Ψ is played by the Wheeler-De Witt equation[12], which is a functional differential equation on superspace. Since one does not know how to solve such an equation, one restricts the infinite number of degrees of freedom of $g_{\mu\nu}$ and ϕ to a finite number; the resulting finite-dimen-sional manifold is called minisuperspace. Here we shall employ a simple minisuperspace model in which we restrict the 3-geometry to be homogeneous, isotropic and closed, so that it is described by a single scale factor a. The scalar field ϕ is restricted to a constant value at one of the extrema of the effective potential, $\phi = \phi_o$. Then the Wheeler-De Witt equation for $\Psi(a)$

takes the form[12,13]

$$\left[a^{-p} \frac{\partial}{\partial a} a^{p} \frac{\partial}{\partial a} - \left(\frac{3\pi}{2G}\right)^2 a^2(1 - H^2a^2)\right] \Psi(a) = 0 \tag{19}$$

Here, the parameter p depends on one's choice of factor ordering. This issue is unimportant for our discussion, and we shall set $p = 0$. (Variation of p affects $\Psi(a)$ only for $a \lesssim G^{1/2}$). Then Eq. (19) takes the form of a one-dimensional Schroedinger equation for a "particle" described by a coordinate $a(t)$, having zero energy and moving in a potential

$$U(a) = \frac{1}{2} \left(\frac{3\pi}{2G}\right)^2 a^2(1-H^2a^2). \tag{20}$$

The WKB solutions of Eq. (19) in the classically allowed region ($a > H^{-1}$) are (disregarding the pre-exponential factor)

$$\Psi_{\pm}^{(1)}(a) = \exp\left(\pm i\int_{H^{-1}}^{a} p(a')da' \mp \frac{i\pi}{4}\right) \tag{21}$$

and the under-barrier ($o < a < H^{-1}$) solutions are

$$\Psi_{\pm}^{(2)}(a) = \exp\left(\pm \int_{a}^{H^{-1}} |p(a')|da'\right), \tag{22}$$

where

$$p(a) = \frac{3\pi}{2G} a(H^2a^2 - 1)^{1/2} \tag{23}$$

Tunneling through the barrier corresponds to the choice of the "outgoing" wave for $a > H^{-1}$: $\Psi(a > H^{-1}) \sim \Psi_{+}^{(1)}(a)$. Then the matching conditions at $a = H^{-1}$ give[14] $\Psi(a < H^{-1}) \sim \Psi_{+}^{(2)}(a)$. The wave function grows exponentially towards $a = 0$ (as it should). The tunneling amplitude is proportional to

$$\exp\left(-\int_{o}^{H^{-1}} |p(a')|da'\right) = \exp\left(-3/16G^2\rho_v\right),$$

and thus the tunneling probability is given by Eq. (3) with $B = 3/8G^2\rho_v$. Note that for $p = -1$, the exact solution of Eq. (19) can be expressed in a closed form in terms of Hankel functions. The solution corresponding to the tunneling boundary condition is $\Psi(a) = \xi^{1/3}H_{1/3}^{(1)}(\xi)$, where $\xi = (\pi H/2G)(a^2-H^{-2})^{3/2}$. Since $\Psi(a)$ is sensitive to p only at $a \lesssim G^{1/2}$, this gives a good approximation for Ψ at $a \gg G^{1/2}$ with any value of p. The semiclassical approximation is justified if $B \gg 1$ or $\rho_v \ll G^{-2}$. This condition is satisfied in most grand unified theories.

5. PATH INTEGRAL APPROACH

The wave function of the universe can be represented in the form of a path integral[13]. In the path integral formulation, the transition amplitude between two 3-geometries, g_1 and g_2, with corresponding scalar field configurations, ϕ_1 and ϕ_2, is given by

$$\int_{(g_1\phi_1)}^{(g_2\phi_2)} [dg_{\mu\nu}][d\phi]\exp(iS[g_{\mu\nu},\phi]), \qquad (24)$$

where S is the action and the integration is over 4-geometries and scalar field histories interpolating between (g_1,ϕ_1) and (g_2,ϕ_2). In the "creation from nothing" picture, the wave function $\Psi(g,\phi)$ is the transition amplitude from "nothing" to the configuration (g,ϕ); this corresponds to shrinking g_1 in Eq. (24) to a point (g_o). Then the wave function of the universe can be written as

$$\Psi(g,\phi) = \int_{g_o}^{(g,\phi)}[dg_{\mu\nu}][d\phi]\exp(iS[g_{\mu\nu},\phi]), \qquad (25)$$

where the integration is over 4-geometries interpolating between g_o and the 3-geometry g with the field configuration ϕ. Since we want the transition amplitude from "nothing" to (g,ϕ), not the other way around, the integration should be done only over the histories lying to the past of (g,ϕ). This corresponds to using Teitelboim's causal propagator for the gravitational field[15].

The definition (25) has a problem in that all Lorentzian 4-geometries connecting g_o and g must be singular. We can restrict them to be nonsingular everywhere except at g_o, but then it is impossible to avoid a singularity at g_o. One can speculate that the causal structure of space-time undergoes a drastic change on scales smaller than the Planck length ($\sim G^{1/2}$), so that the singularity disappears. (This would happen, e.g., in discrete space-time models.) To illustrate this possibility, we can use bubble nucleation as an example. If the bubble starts from zero volume, then the (2+1)-dimensional space-time of the bubble wall is bound to be singular (assuming that we require Lorentzian signature everywhere on the wall). We know, however, that this singularity is unphysical, since the wall cannot be treated as a thin sheet for bubble sizes smaller than the wall thickness, δ. Moreover, the contribution of bubble sizes $R < \delta$ to the action is $\Delta S \sim \sigma\delta^3$, and with B from Eq. (7), $\Delta S/B \sim (\delta/R_o)^3$. Thus, for $R_o >> \delta$ the semiclassical result (7) is not sensitive to the details of the bubble wall structure. Similarly, for $\rho_v << G^{-2}$ we expect that possible modifications of space-time structure on Planck scale will not affect the semiclassical results of this paper.

An alternative possibility, which does not require modifications of general relativity, is to do the integration in (25) over all metrics, including singular ones. Singular metrics with infinite action will not contribute to ψ, but metrics with integrable singularities will. It is easily shown that the class of integrably singular metrics interpolating between g_0 and g is nonempty. A similar approach to the problem of topology change in quantum gravity has been briefly discussed in Ref. 20.

In the semiclassical approximation, the dominant contribution to (25) is given by the classical trajectory and its neighborhood. A classical trajectory connecting g_0 with a given configuration (g,ϕ) may not exist. In fact, if "creation from nothing" is a quantum tunneling process, we expect it not to exist in the classically forbidden region under the barrier. For example, in the simple minisuperspace model of Sec. IV, no classical trajectory passes through a 3-sphere of radius $a < H^{-1}$. To find the under-barrier semiclassical wave function, one has to analytically continue to the integration over Euclidean space-times. (This is similar to what one does in the path-integral approach to non-relativistic quantum mechanics[16]). Then the path integral is dominated by the classical solution of the Euclidean field equations, which, in our case, is the de Sitter instanton (17). With this prescription, the wave function obtained from Eq. (25) is the same as we found in Sec. 4.

Here, I should mention an alternative approach to the definition of the wave function of the universe. Hartle and Hawking[13] have suggested that $\Psi(g,\phi)$ is given by a path integral over all compact <u>Euclidean</u> 4-geometries and scalar field histories bounded by the configuration (g,ϕ):

$$\Psi(g,\phi) = \int [dg_{\mu\nu}][d\phi]\exp(-S_E[g_{\mu\nu},\phi]). \tag{26}$$

Here S_E is the Euclidean action. Although this definition seems to be very similar to ours, the wave function for a de Sitter universe obtained by Hartle and Hawking[13] and by Moss and Wright[17] using Eq. (26) is different from the one obtained here. They find $\Psi(a < H^{-1}) \sim \Psi_-^{(2)}(a)$ and $\Psi(a > H^{-1}) \sim \Psi_+^{(1)}(a) + \Psi_-^{(1)}(a)$ This wave function corresponds to a "particle" bouncing off the potential barrier at $a = H^{-1}$; under the barrier $\Psi(a)$ is exponentially suppressed. It describes a contracting and re-expanding universe. The advantage of the Euclidean approach is that the integration in Eq. (26) can be done over nonsingular compact manifolds, and no modification of the theory on Planck scales is necessary. A serious problem of this apporach is that it automatically gives a time-symmetric picture of the universe: a contracting and re-expanding universe in the case of a de Sitter space and an oscillating universe in more complicated minisuperspace models[13]. In this picture, it is not clear how one can define the arrow of time. Another difficulty with the Euclidean definition of Ψ is mentioned in the next section.

6. IMPLICATIONS AND CONCLUDING REMARKS

Since the wave function can have only a probabilistic interpretation, we are faced with the problem of having only one copy of the universe. How can we then interpret the tunneling probability P?

Eqs. (3) and (16) give

$$P \propto \exp(-3/8G^2\rho_v), \tag{27}$$

where $\rho_v = V(\phi_o)$ and ϕ_o is an extremum of the effective potential. Eq. (27) suggests that of all such extrema, the tunneling is "most probable" to the highest maximum of $V(\phi)$, $\phi = \phi_{max}$. If one assumes the existence of an Observer who can do a statistical survey of all nucleating universes, He will find that most of the universes nucleate with $\phi = \phi_{max}$. [Note that for imaginary creatures living on the bubble walls (see Sec. II), you could be such an observer.] Our best guess seems to be that we live in a "typical" universe which has started with $\phi = \phi_{max}$. It may happen, however, that typical universes are not suitable for life, and then we have to invoke the anthropic principle and conclude that we live in one of the rare universes which nucleated at $\phi \neq \phi_{max}$.

In the same mystical vein, one can create an intuitive picture of the view that opens in front of the Observer studying creation of universes from nothing. To Him, "nothing" is a pure space-time foam[18], without any classical space-time substrate. Most of the bubbles in this "foam" have Planck dimensions. Any bubble of size smaller than H^{-1} collapses with very high probability (this corresponds to the exponential increase of $\Psi(a)$ towards smaller a). Some exceptional bubbles fluctuate to a $\sim H^{-1}$ and make it to the classically allowed region - then a universe is born. This image is somewhat misleading in that it pictures the creation of universes as "happening in time". However, there is no classical time in space-time foam; that is, there is no past and future with causal relations between them. In this sense, "time" is created with the universe.[5] If the maximum at $\phi = \phi_{max}$ is sufficiently flat, the newly born universe will evolve along the lines of a new inflationary scenario[19], as described in Refs. 6, 21. When the vacuum energy eventually thermalizes, the universe heats up to a temperature $T \lesssim \rho^{1/4}$. In our model this is the maximum temperature the universe has ever had.

It should be noted that Hartle-Hawking definition of Ψ, which corresponds to the opposite sign in the exponential (27), predicts that the most probable universes are those corresponding to smallest values of ρ_v. I think this is a serious difficulty of this approach, since small values of ρ_v make inflation and baryosynthesis rather unlikely.

So far, our discussion was based on the classical Einstein action,

$$L_g = -R/16\pi G. \tag{28}$$

At large curvature, $R \gtrsim G^{-1}$, quantum corrections to (28) become more important. These corrections have the form of quadratic and perhaps higher order terms in Riemann tensor. Starobinsky[22] has suggested an inflationary scenario in which the de Sitter phase is obtained as a self-consistent solution of vacuum Einstein's equations modified by the quantum corrections. An extension of our analysis to this case is straightforward. In the quantum version of the Starobinsky scenario the universe tunnels directly to the self-consistent de Sitter phase and the tunneling process is described by the analytic continuation of the de Sitter solution to the Euclidean domain. The tunneling action can be written as $S_E = L_g\Omega$, where $\Omega = 384\pi^2R^{-2}$ is the volume of the 4-sphere (de Sitter instanton). In asymptotically free theories the quantum corrections to L_g due to matter fields approach those due to conformally coupled free fields in the high curvature limit.[23] With these corrections, the gravitational Lagrangian in de Sitter space can be written as

$$L_g = -\frac{1}{16\pi}\left(\frac{R}{G} + \alpha R^2 + \beta R^2 \ln\frac{R}{\mu^2}\right) \qquad (29)$$

The coefficient α can be changed by varying the renormalization scale μ and we shall choose μ so that $\alpha = 0$. The coefficient β, which is related to the trace anomaly, depends on the number of various fields in the model, and for a typical grand unified theory $|\beta| \gtrsim 1$. [If the number of matter fields is large enough, one can assume that quantum corrections due to gravitons can be neglected.] The self-consistent de Sitter solution has $R = (\beta G)^{-1}$, and we find from Eq. (29)

$$S_E = -24\pi\beta[1+\ln(\beta G\mu^2)^{-1}] \qquad (30)$$

Unless there are some unexpected cancellations, Eq. (30) suggests that $|S_E| \gg 1$, and thus the quantum tunneling in the Starobinsky scenario can also be studied in the semiclassical approximation.

Comparing Eqs. (18) and (30) we see that tunneling to a maximum of $V(\phi)$ can compete with that to the Starobinsky phase only if $\rho_V \gtrsim (64\pi\beta G^2)^{-1}$. Thus, "creation from nothing" is likely to produce the initial conditions required in the Starobinsky scenario. (The possibility of creation from nothing in the Starobinsky scenario has been pointed out in Ref. 25.)

The conclusions of the present paper are based on a simple minisuperspace model with only one degree of freedom. Extension to more complicated models would be very interesting. In particular, it would be interesting to calculate the nucleation probability for configurations which are not at the extrema of effective potential. Another interesting direction for future research is "creation from nothing" in Kaluza-Klein theories.

Most of the problems discussed in this paper belong to "metaphysical cosmology", which is the branch of cosmology totally decoupled from observations. This does not mean, however, that such problems do not allow a rational analysis: the ideas can be tested by overall consistency of our picture of the universe. The main advantage of the model presented here is that it gives a cosmological scenario which does not require any initial or boundary conditions. The structure and evolution of the universe(s) are totally determined by the laws of physics.

I would like to note also that the "creation from nothing" picture opens some new theoretical possibilities: it removes a veto from some elementary particle models which are ruled out in the standard inflationary and non-inflationary scenarios. For example, there exists a wide clase of models in which the false vacuum, which is a suitable starting point for inflation, gets destabilized at $T \to \infty$ (that is, it becomes a maximum of the effective potential)[24]. Such inflationary models are ruled out in the hot big bang cosmology, where the universe starts at infinite temperature. In our picture, the initial hot phase is absent, and the universe can tunnel directly to the state at the top of the effective potential.

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